Steps Taken to Implement the Math and Second-Language Minimum Course Requirements of the University of North Carolina (2000)

The University of North Carolina
Office of the President

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Contents

Summary of MCR Implementation Steps

For the Fourth Unit of Mathematics Requirement

1. A review of 1999-2000 course enrollment data revealed that only five public high schools in the state failed to enroll students in a unit of mathematics beyond Algebra II.

2. The same review of course enrollment data revealed that fewer than 10 percent of public high schools failed to offer two or more courses (excluding Computer Math) that would meet the new math requirement. Thus, in over 90 percent of public high schools, students have a choice of courses that they could take to fulfill the requirement of a fourth unit of mathematics.

3. In order to solicit opinions from high school and college mathematics faculty about curriculum revisions that might be needed to address the new requirement, the Math and Science Education Network convened a series of town meetings at eight sites across the State during October and November of 2000. Suggestions from those meetings have led to a proposed restructuring of the mathematics curriculum as outlined in the report.
4. The new curriculum would:
   - Change the title of Advanced Mathematics to Pre-Calculus, since that is the subject taught in the current version of Advanced Mathematics.
   - Create a new course under the old title—Advanced Mathematics—that would cover topics in numeration and numerical operations, advanced algebra and trigonometry, graphing and functional relationships, probability and statistics, spatial sense, measurement and geometry, and mathematical modeling.
   - Revise the content of Discrete Mathematics to distinguish it more clearly from the content of the new Advanced Mathematics course.
   - Continue to offer AP Statistics, AP Calculus AB (& BC), and International Baccalaureate Mathematics as alternatives that will meet the new requirement.

5. A new brochure and other informational materials will be developed to recommend course sequences for two groups of students: those who plan to pursue majors that are quantitative or calculus-based; and those who plan to pursue majors that are non-quantitative.

6. Starting this summer, math department heads will begin to be trained in the curriculum changes. Annual presentations to the NC Council of Teachers of Mathematics will be planned. Teachers will begin to be trained through MSEN summer programs by the summer of 2003. The Standard Course of Study that reflects the curriculum restructuring will be revised by the end of 2002-03.

7. A 21-member steering committee will be appointed to oversee this implementation process.

For the Second-Language Requirement

1. There have been presentations to school counselors, teachers, and statewide education conferences to follow up President Broad's announcement on May 22, 2000 of the new minimum course requirements for undergraduate admission to UNC institutions.

2. Detailed descriptions have been added to the annual *Institutional Profiles* publication.

3. Appropriate changes have been made to the UNC website.

4. Information has been provided through the ***PATHWAYS*** Resource Center and website.

5. The Academic Affairs staff have been meeting with representatives from the Department of Public Instruction (DPI) to discuss: (1) awareness among K-12 educators, students, and parents, and (2) accommodating applicants with special requirements.

6. Certain issues have been discussed with DPI officials:
   - Given that American Sign Language (ASL) has been approved as an acceptable second language, how do campuses currently determine proficiency in ASL?
   - Once admitted (using ASL as the second language), will the student still be required to take a foreign language to meet the campus' graduation requirements?
   - For the student for whom English is a second language, under what conditions will the native language be considered to meet the second language requirement?

   The final decisions on how to demonstrate that students have met the minimum requirements will rest with DPI. We have offered to provide guidance and assistance throughout the process.

7. DPI officials will meet with all the UNC Directors of Admissions at their Fall meeting in Chapel Hill.

8. Dissemination of final solutions will be supported through the ***PATHWAYS*** website and Resource Center.

Steps to Ensure That a Fourth Unit of Mathematics Will Be Available

Current Offerings

In order to ensure that all public high schools are offering a course that will satisfy the 2006 requirements for a fourth unit of mathematics for undergraduate admission to UNC institutions, the N.C. Department of Public Instruction extracted data, by high school, on enrollments in math courses that are beyond the level of Algebra II. The data showed that only five of the 407 public schools in North Carolina with a 12th grade failed to enroll any students in a course that would probably meet the requirement, had it been implemented in 1999 instead of 2006. The courses considered were AP Calculus AB, AP Calculus BC, AP Statistics, IB Math, Discrete Math, Advanced Math (Pre-Calculus), Fifth-Year Math, and Computer Math. These high schools are listed in Category I in Table 1 below.

Ideally, a high school would offer students a choice of courses in fulfilling the new requirement. In order to gauge how many high schools did so in 1999-2000, the N.C. Department of Public Instruction prepared data on the number of high schools that offered at least two courses that would satisfy the requirement. In their analysis, Computer Math—a course not offered by many high schools and one not likely to be included in the final list of courses that will satisfy the requirement—was not counted as one of the two or more courses. The high schools that offered at least two courses other than Computer Math are shown in Category 2.

Table 1. Public High Schools That Did Not Offer Certain Courses in 1999-2000

Category 1. Schools That Did Not Offer a Course in Math beyond Algebra II		
Local Education Agency	**#**	**School**
Macon County	1	Highlands School (students take college algebra and trigonometry at Southwestern CC)
	2	Nantahala School (only 137 students, K-12; a student wanting math could take a fourth course via ITV at Southwestern CC)
Rutherford County	3	R S Central (does provide Algebra III and "advanced math"; students can take still other courses at Isothermal CC)
Swain County	4	Cherokee Extension High
Wake County	5	Green Hope High (will add 12th grade in 2001-02 and offer AB calculus and "advanced geometry")
Category 2[1]. Schools That Did Not Offer at Least Two Math Courses beyond Algebra II (Excluding "Computer Math")		
Local Education Agency	**#**	**School**
Alamance-Burlington	1	Hugh M Cummings High
Alleghany County	2	Alleghany High
Beaufort County	3	Aurora High
	4	Northside High
Camden County	5	Camden County High

[1] Does not include Category 1 schools.

Chatham County	6	Chatham Central High
	7	Northwood High
Cherokee County	8	Hiwassee Dam Union
Columbus County	9	South Columbus High
Cumberland County	10	Massey Hill Classic
Dare County	11	Cape Hatteras Second
Lexington City	12	Lexington Senior High
Durham County	13	Communities in School Academy
Forsyth County	14	Carver High
	15	East Forsyth High
	16	R B Glenn High
	17	North Forsyth High
	18	Reynolds High
Franklin County	19	Franklinton High
Granville County	20	South Granville High
Halifax County	21	Northwest High
Haywood County	22	Central Haywood High
Hertford County	23	Hertford County High
Hoke County	24	J W Turlington
Hyde County	25	Ocracoke
Jackson County	26	Blue Ridge School
Johnston County	27	South Johnston High
Madison County	28	Madison High
Martin County	29	Bear Grass High
	30	Jamesville High
New Hanover County	31	Lakeside
Northhampton County	32	Northhampton High East
Richmond County	33	Richmond Senior High
Robeson County	34	Saint Pauls High
Rockingham County	35	Rockingham County High
Sampson County	36	Hobbton high
Swain County	37	Swain County High
Transylvania County	38	Rosman Middle/High
Vance County	39	Western Vance Second
Washington County	40	Creswell High

Some of the schools in Category 2 may be new schools, or newly consolidated schools, and may not have had the enrollment needed to fill more than one advanced math course in 1999-2000, when the course enrollment counts were taken. Similarly, some of these high schools may have arranged with nearby community colleges or nearby four-year colleges to teach one or more advanced math courses through a dual-enrollment agreement with the high school. In these agreements, course enrollments are counted by the college and not by the high school. For these reasons, the number of high schools in Category 2 errs on the high side. That means

that fewer than 10 percent of public high schools in the state are not offering at least two courses that would meet the fourth-math-course requirement.

New-Course Development

The adoption of the new requirement is giving math curriculum planners an opportunity to develop two sets of fourth-year math courses: one for students who plan to pursue college majors that are quantitative and/or calculus-based, and one for students planning to pursue other college majors. *For students in the first group*, the typical math sequence will include pre-calculus (labeled "Advanced Mathematics" in the State's current course inventory) and calculus (offered as AP Calculus AB and AP Calculus BC, or as a non-AP course). *For students in the second group*, Discrete Math and AP Statistics are the two primary options available to students. However, Discrete Math, for which there is a standard course of study, is offered by only 21 high schools in the State, and AP Statistics is offered by even fewer high schools. Fifth-Year Math, a course for which there is no standard course of study and which includes a varied mix of topics, is offered at some high schools to some students who do not intend to pursue a quantitative or calculus-based major, but its content is locally developed and therefore highly variable.

In an effort to solicit opinion from high school and college mathematics faculty about the content of a new math course that would specifically address the new 2006 requirement, town meetings were conducted in October and November of 2000 at eight sites across the State. Discussion leaders were UNC mathematics faculty. The meetings were organized by the Math/Science Education Network (MSEN) in collaboration with local school districts and the N.C. Department of Public Instruction. The focus of the discussion was on the specific topics that should be included in a new course that would require Algebra II as a prerequisite, but would not be part of a math sequence that would prepare students for quantitative or calculus-based majors.

The ideas generated by these meetings are to be summarized for high school math teachers statewide in their magazine *Centroid*, and will be the focus of discussion this summer when high school mathematics department heads attend workshops sponsored by DPI curriculum planners. Following the review and synthesis that will follow these workshops, a draft proposal will be taken to the fall meeting of the N.C. Council of Teachers of Mathematics (NCCTM).

The fundamental approach being followed in this curriculum restructuring is to develop two course alternatives in addition to AP Statistics and IB for college-bound students who are not planning to pursue a calculus-based major. *The first step* in this process will be to clarify the content of the current Advanced Mathematics course and change its title to "Pre-Calculus." This will be the course that will fall between Algebra II and Calculus for those students who will pursue a calculus-based major. *The second step* will be to develop a totally new course under the old label Advanced Mathematics that will cover topics suggested in the fall 2000 MSEN town meetings (Table 5 in Appendix B. Concepts to Be Covered in New Math Courses, p. 14). *The final step* will be to rework the current course in Discrete Mathematics to include topics shown in the appendix (Table 4, p. 13) the outline of course skills. These topics include advanced algebra, trigonometry, modeling, and probability and statistics. A Standard Course of Study will be developed for each course by the end of the 2002-03 academic year. Because

both the Discrete Mathematics and the Advanced Mathematics courses will have modules that cover topics in probability, statistics, and other topics that use algebra, they will reinforce algebra skills, so that students will be prepared to start college mathematics courses at a non-remedial level. Table 2 summarizes these changes.

Table 2. Model Math Course Sequences for College-Bound High School Students

A. Students Planning to Major in a Quantitative Field	1. Algebra I, Algebra II, Geometry, and: a. Pre-Calculus, Calculus. . . b. Advanced Algebra/Trigonometry, Pre-Calculus, Calculus
	2. Integrated Mathematics I, II, and III, and: a. Pre-Calculus, Calculus. . . b. Advanced Algebra/Trigonometry, Pre-Calculus, Calculus
B. Students Planning to Major in a Non-Quantitative Field	1. Algebra I, Algebra II, Geometry, and: a. Discrete Mathematics b. Advanced Mathematics c. AP Statistics
	2. Integrated Mathematics I, II, and III, and: a. Discrete Mathematics b. Advanced Mathematics c. AP Statistics

Schedule

A schedule for these course-development activities is shown in Table 3. Note that the schedule calls for the development of training materials for teachers that will be used by DPI and MSEN in training sessions planned to commence in the 2002-03 academic year. Coincident with the development of these materials will be the creation of a new brochure for students and parents that will summarize the course options for college-bound students and the relationship of these options to students' planned majors. The content of the brochure will also be summarized in ***PATHWAYS*** content; in UNC's *Institutional Profiles*, a publication developed specifically for high school counselors; and in materials used in the Early Math Placement Testing (EMPT) program.

Table 3. Development Schedule for Fourth Math Course

Oct/Nov 2000	Math/Science Education Network (MSEN) conducts eight town meetings on the content needed in a math course. Discussion leaders are Ralph Devane (WCU) and David Royster (UNCC). Invited participants are high school and college math teachers.
Feb. 20, 2001	Meeting to plan next steps and identify members of an implementation committee.
Spring 2001	Article summarizing the recommendations of the MSEN Town Meetings is submitted to *Centroid*, the official trade magazine for NC high school teachers of mathematics.
Apr. 20, 2001	DPI math curriculum committee and MSEN staff meet in Asheville to plan summer course development activities and refine content of new course(s).
Jun. 4, 2001	Meeting with math curriculum leaders to settle on an implementation schedule and a list of persons to serve on a statewide implementation steering committee. Representatives to be drawn from DPI, MSEN, NCSSM, EMPT Advisory Committee, NCCCS, UNC OP, LEAs, school/college faculty, and DPI Testing/Assessment staff.
Summer 2001	DPI sponsored summer institute for department chairs in high school mathematics departments reviews content of new course(s).
Oct. 2001	Presentations at NCCTM state conference, and annually thereafter.

2001-03	DPI drafts Standard Course of Study for new course(s) and develops training materials for teachers of the new course(s).
2002-03	DPI and MSEN together develop draft of training materials.
2002-05	MSEN offers summer training institutes for teachers of the new course(s).

Related Issues

State standards currently require that courses contained in the UNC Minimum Course Requirements should have end-of-course (EOC) tests in order to reinforce the course content in the Standard Course of Study. Unfortunately, DPI resources have not been sufficient to develop all of these tests, and the creation of additional math courses to meet the fourth-unit requirement will widen this gap by four courses. The matter is complicated by legislation likely to pass the current legislative session that will require the University of North Carolina to study the role of EOC tests in the admissions and course-placement process. Since one would expect that an EOC test score in a student's highest-level and most current math course is likely to be a better predictor of college academic performance than an EOC test score in a prerequisite course, there is likely to be added pressure to create EOC tests for courses that satisfy the fourth-unit requirement. The issue of EOC tests for the new mathematics courses will need to be resolved in future reports.

In order to ensure continuity of effort and oversight, it will be necessary to appoint a Steering Committee on Implementation of the 2006 MCR Mathematics Course Requirement. The committee should contain representatives from the North Carolina Department of Public Instruction; college and university mathematics faculty; the University's EMPT program; the secondary schools; teacher training organizations such as MSEN, the Teacher Academy, and the College Board; secondary school counselors; the North Carolina School of Science and Mathematics; and others who might be necessary to ensure full representation in the implementation process.

Steps to Implement the Second-Language Requirement

Effective in Fall 2004 and Fall 2005. . . Six course units in language, including

- four units in English emphasizing grammar, composition, and literature, and
- **two units of a language other than English**

Implementation Plan:

Following the announcement from President Broad's office to the education community (K-12 and higher education), business community, and professional associations (the text of the letter is included in Appendix A. Text of President Broad's MCR Announcement, p. 11), there have been follow-up presentations to school counselors, teachers, and statewide education conferences. In addition, detailed descriptions have been added to the annual *Institutional Profiles* publication. Changes have also been made to the appropriate portions of the UNC website, and information is now provided through the Pathways Resource Center and website.

Meetings with DPI for Planning:

The Academic Affairs staff have been meeting with representatives from DPI to discuss: (1) awareness among K-12 educators, students, and parents, and (2) accommodating applicants with special requirements.

In a recent meeting, DPI officials met with Academic Affairs staff and four admissions directors (UNCC, UNCG, UNCCH, NCSU) where foreign language is an additional requirement by the campus today. The following issues were discussed:

- American Sign Language (ASL) has been approved as an acceptable second language. How do campuses currently determine proficiency in ASL?
- Once admitted (using ASL as the second language), will the student still be required to take a foreign language to meet the campus' graduation requirements?
- For the student for whom English is a second language, under what conditions will the native language be considered to meet the second language requirement?

Numerous points and suggestions were raised in the meeting for the DPI staff to consider. The final decisions on how to demonstrate that students have met the minimum requirements will rest with DPI. However, the university has willingly agreed to assist in providing guidance and any help possible throughout the process.

Next Steps:

- DPI officials will meet with all the UNC Directors of Admissions at their Fall meeting in Chapel Hill.
- Dissemination of final solutions will be supported through the Pathways website and resource center.

Appendix A. Text of President Broad's MCR Announcement

Memorandum

To: NC School District Superintendents
NC Principals and School Counselors in all public, private, and federal schools with grades 7, 8…, or 12
NC Education Cabinet members
NC Board of Education members
NC Board of Community Colleges members
NC School Boards Association members
NC Public School Forum members
NC Association of Educators members
Professional Educators of NC organization
Parents for the Advancement of Gifted Education organization
NC Association of School Administrators
NC School Counselor Association
NC Council of Teachers of Mathematics
Foreign Language Association of NC
NC Christian School Association
North Carolinians for Home Education organization
NC Business Committee for Education
NC Congress of Parents and Teachers, Inc. (PTA)

cc: UNC Board of Governors members
UNC Chancellors
UNC Chief Academic Officers
UNC Directors of Admissions

From: Molly Corbett Broad

Date: May 22, 2000

Re: Implementation of the Increased Minimum Course Requirements for Undergraduate Admission to a UNC Institution

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In an effort to ensure that every student entering a UNC institution has a strong chance of successfully completing a baccalaureate degree, the UNC Board of Governors voted unanimously last month to increase the minimum course requirements needed for admission to any UNC institution. The specific actions taken were to require:

- Two units of a language other than English, effective in the fall semester of 2004; and
- One additional unit of mathematics beyond algebra II, raising the total number of required units in mathematics to four, effective in the fall semester of 2006. *This higher math requirement will affect applicants to all institutions except the North Carolina School of the Arts.*

By these actions, the Board has moved from *recommending* that students take at least two units of a language other than English and a fourth unit of mathematics to *requiring* that all applicants to UNC institutions do so by the dates set forth above to be considered for acceptance.

The Board's action is based on a growing body of research evidence that successful completion of these courses is strongly related to academic success in college. A study of the relationship between course-taking patterns of 1997-98 public high school seniors in North Carolina and their subsequent freshman year academic performance in UNC institutions revealed that students who had taken two or more units of the same foreign language and the fourth unit of mathematics performed significantly better than those who met only the current minimum course requirements. Measures of comparative performance included SAT scores, placement in remedial courses, first-year GPA, and freshman-to-sophomore retention. In addition, an independent national study of high school curricula found that students who completed a fourth year of mathematics more than doubled their chance of graduating from college. These findings are presented in a report prepared for the Board of Governors entitled *Background on the Increase in the UNC Board of Governors' Minimum Course Requirements for Undergraduate Admission.* It is available on the University of North Carolina website at *http://ga.unc.edu/UNCGA/assessment/*, or you may order a paper copy by contacting Morris Dean, Division of Program Assessment and Public Service, UNC General Administration, P.O. Box 2688, Chapel Hill, NC 27515-2688, (919)962-4597, *mdean@ga.unc.edu*.

In taking action to raise course requirements, the Board recognized that it would need your help in notifying students, their parents, and teachers of the changes as soon as possible. It also recognized that the Office of the President would need to work closely with our many partners in education to ensure that courses, teachers, student and parent information, curriculum alignment, and teacher/counselor training needed for successful implementation of the new requirements would be provided. We are committed to an unprecedented level of collaboration to do these things, and we hope you will share your thoughts on where you think our involvement will be most helpful. We begin this effort by notifying you of these changes. We will follow immediately with a plan to provide information about these changes to every student and parent directly.

In addition to these initiatives, we will work with the N.C. Department of Public Instruction to produce an annual report on student course-taking patterns in the public high schools in order to verify that high school classes of 2004 and 2006 are on track to meet the higher requirements. Finally, we plan to make extensive use of a new student guidance initiative called *PATHWAYS* to inform students and their parents of the new requirements and to encourage students to meet them even before they are required.

As you work to inform students and others of these requirements, please be careful to note that individual constituent institutions may have higher course requirements for admission than the systemwide *minimum* course requirements. Of course, they will all have other non-course requirements too. Therefore, you should be careful to note that meeting the minimum course requirements set for 2004 and 2006 will *not*—either then or now—*assure* admission to any given constituent institution. Indeed, these are *minimum* course requirements. Finally, for your information, we have enclosed a sheet that answers some of the questions that are already being asked about the revised requirements. If you have others, please feel free to forward them to Mary Wakeford, Interim Associate Vice President for Academic Affairs, (919)962-4613.

We thank you for your support and assistance.

[signed by Molly Corbett Broad]

Appendix B. Concepts to Be Covered in New Math Courses

Table 4. Discrete Mathematics: Outline of Concepts to Be Covered

Discrete Mathematics introduces students to the mathematics of networks, social choice, and decision making. The course extends students' application of matrix arithmetic and probability. Applications and modeling are central to this course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.
Number Sense, Numeration, and Numerical Operations **Goal: The learner will solve problems involving social choice and decision making.**
1.01 Solve problems involving election methods: Plurality, Run-off, Sequential Run-off, Borda, Condorcet. 1.02 Solve problems involving weighted voting, voting power, and winning coalitions. 1.03 Solve problems involving estate division. 1.04 Solve problems involving continuous fair division. 1.05 Solve problems involving apportionment: including, but not limited to, Hamilton, Jefferson, Hill, Webster, Adams, Quota (Balinski and Young).
Spatial Sense, Measurement, and Geometry **Goal: The learner will use graphs to solve problems.**
2.01 Define the terms associated with a graph (edges, vertices, degree, paths, circuits, connected, disconnected, and trees). 2.02 Represent problem situations using finite graphs and adjacency matrices. 2.03 Find the critical path(s) using PERT (Program and Evaluation Review Technique) 2.04 Find an Euler circuit or path, if it exists. 2.05 Find a Hamiltonian circuit or path, if it exists. 2.06 Solve problems involving Euler and Hamiltonian circuits. 2.07 Find the minimum-cost spanning tree for a given graph. 2.08 Incorporate graph coloring to solve real-world problems using the four color theorem and chromatic numbers. 2.09 Use binary expression trees to solve problems in Polish and reverse Polish notation. 2.10 Solve problems involving bin packing.
Patterns, Relationships, and Functions **Goal: The learner will use matrices, functions, sequences, and series to solve problems.**
3.01 Solve problems requiring matrix operations. a) Solve linear systems through applications (Leontief Input-Output Model). b) Solve problems involving communication networks. c) Use transition matrices, such as Leslie Matrix and Markov Chains, to make predictions d) Use matrices to produce coordinate transformations. e) Use matrices to determine harvesting strategies to stabilize a population. 3.02 Use recursive relations to solve problems. 3.03 Verify explicit (closed-form) definitions using mathematical induction. 3.04 Find explicit (closed form) definitions using finite differences and geometric or arithmetic formulas. 3.05 Use mixed recursion to solve problems involving growth and decay. 3.06 Use sequences and series to solve problems. a) Find the sum of a finite sequence. b) Find the sum of an infinite sequence. c) Determine if a given series converges or diverges.

d) Represent a series by using sigma notation.
Data, Probability, and Statistics **Goal: The learner will solve problems involving counting and probability.**
4.01 Use Venn diagram to solve counting problems involving intersection and union of sets. 4.02 Use basic laws of logic to solve more complicated Venn diagram problems. 4.03 Solve problems using addition and multiplication principles. 4.04 Solve problems involving permutations and combinations, including independent., dependent, mutually exclusive, and circular. 4.05 Use experimental probability and simulations for probability models. 4.06 Find expected values and determine fairness. 4.07 Identify discrete random variables and use them to solve problems. 4.08 Derive and apply the Binomial Probability Theorem.

Table 5. Advanced Mathematics: Outline of Concepts to Be Covered

Advanced Mathematics provides students a complete study of trigonometry, as well as advanced algebra topics, analytic geometry, sequences and series, and data analysis. Applications and modeling should be included throughout the course of study. Appropriate technology, from manipulatives to calculators and application software, should be used regularly for instruction and assessment.
Number Sense, Numeration, and Numerical Operations **Goal: The student will perform operations with numbers and vectors and translate between coordinate systems.**
1.01 Convert points in two dimensions between rectangular and polar coordinate systems. 1.02 Operate with vectors in two and three dimensions to solve problems. a) Add and subtract vectors; multiply vectors by a scaler. b) Define and find the inner product of vectors. c) Express vectors as the sum of unit vectors. 1.03 Convert complex numbers between rectangular and polar forms; use DeMoivre's Theorem to find roots and powers of complex numbers.
Spatial Sense, Measurement, and Geometry **Goal: The learner will use trigonometric relationships and transformations to solve problems.**
2.01 Develop and use the trigonometric relationships to solve problems. a) Determine the values of sine and cosine as represented on the unit circle; include multiples of $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, and π b) Find the values of other trigonometric relationships when given the value of one trigonometric relationship. c) Use the unit circle to develop, recognize, and validate trigonometric identities. d) Identify the relationship between trigonometry in degree mode and trigonometry in radian mode. e) Find the radian measure that corresponds to a given angle or arc length. 2.02 Develop and use the Law of Sines and Law of Cosines to solve problems involving triangles and vectors. 2.03 Use coordinate geometry to describe solid figures. a) Identify the coordinates of the vertices of polyhedra. b) Transform polygons in space; describe the results. c) Transform polygons and polyhedra; use matrix operations to describe the transformation.

Patterns, Relationships, and Functions
Goal: The learner will use relations and functions to solve problems.

3.01 Graph and use the basic functions (constant, linear, quadratic, cubic, square root, absolute value, reciprocal, rational, trigonometric, exponential, logarithmic, piecewise defined, and greatest integer) to solve problems.
- a) Compare information given by local behavior versus global behavior.
- b) Determine the symmetry of a given graph.
- c) Identify continuous and discontinuous functions and locate points of discontinuity.
- d) Graph transformations and combinations of transformations for all the functions.
- e) Find coordinates of maximum or minimum points of a given function.
- f) Write the equation of a function given a set of data or other descriptions of its behavior.
- g) Solve equations and inequalities; justify steps used.
- h) Compose two functions and find the domain of the composition.
- i) Analyze a function by decomposing it into simpler functions.
- j) Find the inverse of a function and the domain of the inverse.

3.02 Graph and use the basic quadratic relations (parabola, circle, ellipse, hyperbola) to solve problems.
- a) Compare information given by local behavior versus global behavior.
- b) Determine the symmetry of a given graph.
- c) Identify continuous and discontinuous relations and locate points of discontinuity.
- d) Graph transformations and combinations of transformations for all the relations.
- e) Find coordinates of maximum or minimum points of a given relation.
- f) Write the equation of a relation given a set of data, characteristics, or other descriptions of its behavior.
- g) Solve equations and inequalities; justify steps used.
- h) Analyze and graph a relation by decomposing it into simpler relations.
- i) Find the inverse of a relation and the domain of the inverse.

3.03 Use trigonometric and inverse trigonometric functions to solve problems.
- a) Express the tangent, cotangent, secant, and cosecant functions in terms of sine and cosine.
- b) Sketch a graph of each of the six trigonometric functions and identify the period of each.
- c) Recognize and graph transformations of each of the six trigonometric functions.
- d) Use graphs to develop, recognize, and validate trigonometric identities.
- e) Solve trigonometric equations and inequalities; justify steps used.
- f) Find values of inverse trigonometric functions, applying appropriate domain and range restrictions.
- g) Evaluate and graph compositions of trigonometric and inverse trigonometric functions.

3.04 Use polar equations to solve problems.
- a) Graph polar equations; identify transformations related to changes in constants and coefficients.
- b) Translate quadratic relations between rectangular and polar (parametric) forms; graph.
- c) Graph, and model real world phenomena using parametric equations.

Data, Probability, and Statistics
Goal: The learner will create and use models of data for reporting and analysis.

4.01 Use sequences and series to solve problems.
- a) Find indicated terms in sequences.
- b) Use summative notation to describe the sums in a series.
- c) Find the sum of a finite series and of an infinite geometric series.
- d) Find the limit of an infinite sequence.
- e) Find whether a given series converges or diverges.

(Continued)

4.02 Create and use mathematical models of linear, polynomial, exponential, trigonometric, power, and logarithmic functions to solve problems.

a) Linearize data using concepts of composition and inverses in order to find a model for data. Rewrite the linear equation that models linearized data to fit the original curved data.

b) Model growth and decay using recursive relations; compare with $y = abx$ and $y = (1 + r)x$ forms.

c) Use trigonometric functions to model periodic phenomena.

d) Find the model of the curve of best-fit (linear, polynomial, exponential, power, logarithmic, and logistic) for a set of data.

e) Interpret constants, coefficients, and bases in the context of the data being modeled.

f) Check the model for goodness-of-fit and use the model, where appropriate, to draw conclusions or make predictions.

4.03 Summarize distributions of univariate data to solve problems.

a) Determine measures of central tendency (median, mean) and spread (range, standard deviation).

b) Identify data by its position in the distribution (quartiles, percentiles).

c) Recognize, define, and use the normal distribution curve.